

Useful Inequalities $\{x^2 \geq 0\}$ v0.30a · February 14, 2018

Cauchy-Schwarz $\left(\sum_{i=1}^n x_i y_i\right)^2 \leq \left(\sum_{i=1}^n x_i^2\right) \left(\sum_{i=1}^n y_i^2\right)$

Minkowski $\left(\sum_{i=1}^n |x_i + y_i|^p\right)^{\frac{1}{p}} \leq \left(\sum_{i=1}^n |x_i|^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^n |y_i|^p\right)^{\frac{1}{p}}$ for $p \geq 1$.

Hölder $\sum_{i=1}^n |x_i y_i| \leq \left(\sum_{i=1}^n |x_i|^p\right)^{1/p} \left(\sum_{i=1}^n |y_i|^q\right)^{1/q}$ for $p, q > 1$, $\frac{1}{p} + \frac{1}{q} = 1$.

Bernoulli $(1+x)^r \geq 1+rx$ for $x \geq -1$, $r \in \mathbb{R} \setminus (0, 1)$. Reverse for $r \in [0, 1]$.

$(1+x)^r \leq 1+(2^r-1)x$ for $x \in [0, 1]$, $r \in \mathbb{R} \setminus (0, 1)$.

$(1+x)^n \leq \frac{1}{1-nx}$ for $x \in [-1, 0]$, $n \in \mathbb{N}$.

$(1+x)^r \leq 1 + \frac{rx}{1-(r-1)x}$ for $x \in [-1, \frac{1}{r-1}]$, $r > 1$.

$(1+nx)^{n+1} \geq (1+(n+1)x)^n$ for $x \in \mathbb{R}$, $n \in \mathbb{N}$.

$xy > \frac{x}{x+y}$ for $x > 0$, $y \in (0, 1)$.

$(a+b)^n \leq a^n + nb(a+b)^{n-1}$ for $a, b \geq 0$, $n \in \mathbb{N}$.

exponential $e^x \geq (1+\frac{x}{n})^n \geq 1+x$, $(1+\frac{x}{n})^n \geq e^x(1-\frac{x^2}{n})$ for $n > 1$, $|x| \leq n$.

$e^x \geq e^x$ for $x \in \mathbb{R}$, and $\frac{x^n}{n!} + 1 \leq e^x \leq (1+\frac{x}{n})^{n+x/2}$ for $x, n > 0$.

$e^x \geq 1+x+\frac{x^2}{2}$ for $x \geq 0$, reverse otherwise.

$a^x \leq 1+(a-1)x$ and $a^{-x} \leq 1-\frac{(a-1)}{a}x$ for $x \in [0, 1]$, $a \geq 1$.

$\frac{1}{2-x} < x^x < x^2-x+1$ for $x \in (0, 1]$.

$x^{1/r}(x-1) \leq rx(x^{1/r}-1)$ for $x, r \geq 1$.

$xy+y^x > 1$ and $e^x > (1+\frac{x}{y})^y > e^{\frac{xy}{x+y}}$ for $x, y > 0$.

$2-y-e^{-x-y} \leq 1+x \leq y+e^{x-y}$, and $e^x \leq x+e^{x^2}$ for $x, y \in \mathbb{R}$.

$(1+\frac{x}{p})^p \geq (1+\frac{x}{q})^q$ for $(i) x > 0$, $p > q > 0$,

(ii) $-p < -q < x < 0$, (iii) $-q > -p > x > 0$. Reverse for:

(iv) $q < 0 < p$, $-q > x > 0$, (v) $q < 0 < p$, $-p < x < 0$.

logarithm $\frac{x-1}{x} \leq \ln(x) \leq \frac{x^2-1}{2x} \leq x-1$, $\ln(x) \leq n(x^{\frac{1}{n}}-1)$ for $x, n > 0$.

$\frac{2x}{2+x} \leq \ln(1+x) \leq \frac{x}{\sqrt{x+1}}$ for $x \geq 0$, reverse for $x \in (-1, 0]$.

$\ln(n+1) < \ln(n) + \frac{1}{n} \leq \sum_{i=1}^n \frac{1}{i} \leq \ln(n) + 1$

$\ln(1+x) \geq \frac{x}{2}$ for $x \in [0, \sim 2.51]$, reverse elsewhere.

$\ln(1+x) \geq x - \frac{x^2}{2} + \frac{x^3}{4}$ for $x \in [0, \sim 0.45]$, reverse elsewhere.

$\ln(1-x) \geq -x - \frac{x^2}{2} - \frac{x^3}{2}$ for $x \in [0, \sim 0.43]$, reverse elsewhere.

trigonometric $x - \frac{x^3}{2} \leq x \cos x \leq \frac{x \cos x}{1-x^2/3} \leq x \sqrt[3]{\cos x} \leq x - x^3/6 \leq x \cos \frac{x}{\sqrt{3}} \leq \sin x$,

hyperbolic $x \cos x \leq \frac{x^3}{\sinh^2 x} \leq x \cos^2(x/2) \leq \sin x \leq (x \cos x + 2x)/3 \leq \frac{x^2}{\sinh x}$,

$\max\left\{\frac{2}{\pi}, \frac{\pi^2-x^2}{\pi^2+x^2}\right\} \leq \frac{\sin x}{x} \leq \cos \frac{x}{2} \leq 1 \leq 1 + \frac{x^2}{3} \leq \frac{\tan x}{x}$ for $x \in [0, \frac{\pi}{2}]$.

square root

$2\sqrt{x+1} - 2\sqrt{x} < \frac{1}{\sqrt{x}} < \sqrt{x+1} - \sqrt{x-1} < 2\sqrt{x} - 2\sqrt{x-1}$ for $x \geq 1$.

$1 - \frac{x}{2} - \frac{x^2}{2} \leq \sqrt{1-x} \leq 1 - \frac{x}{2}$.

binomial

$\max\left\{\frac{n^k}{k!}, \frac{(n-k+1)^k}{k!}\right\} \leq \binom{n}{k} \leq \frac{n^k}{k!} \leq \left(\frac{en}{k}\right)^k$ and $\binom{n}{k} \leq \frac{n^n}{k^k(n-k)^{n-k}} \leq 2^n$.

$\frac{n^k}{4k!} \leq \binom{n}{k}$ for $\sqrt{n} \geq k \geq 0$ and $\frac{4^n}{\sqrt{\pi n}}(1-\frac{1}{8n}) \leq \binom{2n}{n} \leq \frac{4^n}{\sqrt{\pi n}}(1-\frac{1}{9n})$.

$\binom{n_1}{k_1} \binom{n_2}{k_2} \leq \binom{n_1+n_2}{k_1+k_2}$ for $n_1 \geq k_1 \geq 0$, $n_2 \geq k_2 \geq 0$.

$\frac{\sqrt{\pi}}{2} G \leq \binom{n}{\alpha n} \leq G$ for $G = \frac{2^{nH(\alpha)}}{\sqrt{2\pi n\alpha(1-\alpha)}}$, $H(x) = -\log_2(x^x(1-x)^{1-x})$.

$\sum_{i=0}^d \binom{n}{i} \leq \min\{n^d+1, (\frac{en}{d})^d, 2^n\}$, for $n \geq d \geq 1$.

$\sum_{i=0}^{\alpha n} \binom{n}{i} \leq \min\left\{\frac{1-\alpha}{1-2\alpha} \binom{n}{\alpha n}, 2^n \cdot \exp(-2n(\frac{1}{2}-\alpha)^2)\right\}$ for $\alpha \in (0, \frac{1}{2})$.

Stirling

$e\left(\frac{n}{e}\right)^n \leq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/(12n+1)} \leq n! \leq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/12n} \leq en\left(\frac{n}{e}\right)^n$

means

$\min x_i \leq \frac{n}{\sum x_i^{-1}} \leq (\prod x_i)^{1/n} \leq \frac{1}{n} \sum x_i \leq \sqrt{\frac{1}{n} \sum x_i^2} \leq \frac{\sum x_i^2}{\sum x_i} \leq \max x_i$

power means

$M_p \leq M_q$ for $p \leq q$, where $M_p = (\sum_i w_i |x_i|^p)^{1/p}$, $w_i \geq 0$, $\sum_i w_i = 1$.

In the limit $M_0 = \prod_i |x_i|^{w_i}$, $M_{-\infty} = \min_i \{x_i\}$, $M_{\infty} = \max_i \{x_i\}$.

Lehmer

$\frac{\sum_i w_i |x_i|^p}{\sum_i w_i |x_i|^{p-1}} \leq \frac{\sum_i w_i |x_i|^q}{\sum_i w_i |x_i|^{q-1}}$ for $p \leq q$, $w_i \geq 0$.

log mean

$\sqrt{xy} \leq \left(\frac{\sqrt{x}+\sqrt{y}}{2}\right) (xy)^{\frac{1}{4}} \leq \frac{x-y}{\ln(x)-\ln(y)} \leq \left(\frac{\sqrt{x}+\sqrt{y}}{2}\right)^2 \leq \frac{x+y}{2}$ for $x, y > 0$.

Heinz

$\sqrt{xy} \leq \frac{x^{1-\alpha}y^\alpha + x^\alpha y^{1-\alpha}}{2} \leq \frac{x+y}{2}$ for $x, y > 0$, $\alpha \in [0, 1]$.

Maclaurin-Newton

$S_k^2 \geq S_{k-1}S_{k+1}$ and $\sqrt[k]{S_k} \geq \sqrt[k+1]{S_{k+1}}$ for $1 \leq k < n$,
 $S_k = \frac{1}{\binom{n}{k}} \sum_{1 \leq i_1 < \dots < i_k \leq n} a_{i_1} a_{i_2} \dots a_{i_k}$, and $a_i \geq 0$.

Jensen

$\varphi(\sum_i p_i x_i) \leq \sum_i p_i \varphi(x_i)$ where $p_i \geq 0$, $\sum p_i = 1$, and φ convex.

Alternatively: $\varphi(E[X]) \leq E[\varphi(X)]$. For concave φ the reverse holds.

Chebyshev

$\sum_{i=1}^n f(a_i)g(b_i)p_i \geq \left(\sum_{i=1}^n f(a_i)p_i\right)\left(\sum_{i=1}^n g(b_i)p_i\right) \geq \sum_{i=1}^n f(a_i)g(b_{n-i+1})p_i$

for $a_1 \leq \dots \leq a_n$, $b_1 \leq \dots \leq b_n$ and f, g nondecreasing, $p_i \geq 0$, $\sum p_i = 1$.

Alternatively: $E[f(X)g(X)] \geq E[f(X)]E[g(X)]$.

rearrangement

$\sum_{i=1}^n a_i b_i \geq \sum_{i=1}^n a_i b_{\pi(i)} \geq \sum_{i=1}^n a_i b_{n-i+1}$ for $a_1 \leq \dots \leq a_n$,

$b_1 \leq \dots \leq b_n$ and π a permutation of $[n]$. More generally:

$\sum_{i=1}^n f_i(b_i) \geq \sum_{i=1}^n f_i(b_{\pi(i)}) \geq \sum_{i=1}^n f_i(b_{n-i+1})$

with $(f_{i+1}(x) - f_i(x))$ nondecreasing for all $1 \leq i < n$.

Weierstrass $\prod_i (1-x_i)^{w_i} \geq 1 - \sum_i w_i x_i$ where $x_i \leq 1$, and either $w_i \geq 1$ (for all i) or $w_i \leq 0$ (for all i).
If $w_i \in [0, 1]$, $\sum w_i \leq 1$ and $x_i \leq 1$, the reverse holds.

Kantorovich $(\sum_i x_i^2)(\sum_i y_i^2) \leq \left(\frac{A}{G}\right)^2 (\sum_i x_i y_i)^2$ for $x_i, y_i > 0$,
 $0 < m \leq \frac{x_i}{y_i} \leq M < \infty$, $A = (m+M)/2$, $G = \sqrt{mM}$.

sum & integral $\int_{L-1}^U f(x) dx \leq \sum_{i=L}^U f(i) \leq \int_L^{U+1} f(x) dx$ for f nondecreasing.

Cauchy $\varphi'(a) \leq \frac{f(b)-f(a)}{b-a} \leq \varphi'(b)$ where $a < b$, and φ convex.

Hermite $\varphi\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b \varphi(x) dx \leq \frac{\varphi(a)+\varphi(b)}{2}$ for φ convex.

Gibbs $\sum_i a_i \log \frac{a_i}{b_i} \geq a \log \frac{a}{b}$ for $a_i, b_i \geq 0$, or more generally:
 $\sum_i a_i \varphi\left(\frac{b_i}{a_i}\right) \leq a \varphi\left(\frac{b}{a}\right)$ for φ concave, and $a = \sum a_i$, $b = \sum b_i$.

Chong $\sum_{i=1}^n \frac{a_i}{a_{\pi(i)}} \geq n$ and $\prod_{i=1}^n a_i^{a_i} \geq \prod_{i=1}^n a_i^{a_{\pi(i)}}$ for $a_i > 0$.

Schur $x^t(x-y)^k(x-z)^k + y^t(y-z)^k(y-x)^k + z^t(z-x)^k(z-y)^k \geq 0$
where $x, y, z, t, k \geq 0$.

Young $\left(\frac{1}{px^p} + \frac{1}{qy^q}\right)^{-1} \leq xy \leq \frac{x^p}{p} + \frac{y^q}{q}$ for $x, y \geq 0$, $p, q > 0$, $\frac{1}{p} + \frac{1}{q} = 1$.

Shapiro $\sum_{i=1}^n \frac{x_i}{x_{i+1}+x_{i+2}} \geq \frac{n}{2}$ where $x_i > 0$, $(x_{n+1}, x_{n+2}) := (x_1, x_2)$,
and $n \leq 12$ if even, $n \leq 23$ if odd.

Hadamard $(\det A)^2 \leq \prod_{i=1}^n \sum_{j=1}^n A_{ij}^2$ where A is an $n \times n$ matrix.

Schur $\sum_{i=1}^n \lambda_i^2 \leq \sum_{i,j=1}^n A_{ij}^2$ and $\sum_{i=1}^k d_i \leq \sum_{i=1}^k \lambda_i$ for $1 \leq k \leq n$.
 A is an $n \times n$ matrix. For the second inequality A is symmetric.
 $\lambda_1 \geq \dots \geq \lambda_n$ the eigenvalues, $d_1 \geq \dots \geq d_n$ the diagonal elements.

Ky Fan $\frac{\prod_{i=1}^n x_i^{a_i}}{\prod_{i=1}^n (1-x_i)^{a_i}} \leq \frac{\sum_{i=1}^n a_i x_i}{\sum_{i=1}^n a_i (1-x_i)}$ for $x_i \in [0, \frac{1}{2}]$, $a_i \in [0, 1]$, $\sum a_i = 1$.

Aczél $(a_1 b_1 - \sum_{i=2}^n a_i b_i)^2 \geq (a_1^2 - \sum_{i=2}^n a_i^2)(b_1^2 - \sum_{i=2}^n b_i^2)$
given that $a_1^2 > \sum_{i=2}^n a_i^2$ or $b_1^2 > \sum_{i=2}^n b_i^2$.

Mahler $\prod_{i=1}^n (x_i + y_i)^{1/n} \geq \prod_{i=1}^n x_i^{1/n} + \prod_{i=1}^n y_i^{1/n}$ where $x_i, y_i > 0$.

Abel $b_1 \min_k \sum_{i=1}^k a_i \leq \sum_{i=1}^n a_i b_i \leq b_1 \max_k \sum_{i=1}^k a_i$ for $b_1 \geq \dots \geq b_n \geq 0$.

Milne $(\sum_{i=1}^n (a_i + b_i)) \left(\sum_{i=1}^n \frac{a_i b_i}{a_i + b_i}\right) \leq (\sum_{i=1}^n a_i) (\sum_{i=1}^n b_i)$

Carleman $\sum_{k=1}^n \left(\prod_{i=1}^k |a_i|\right)^{1/k} \leq e \sum_{k=1}^n |a_k|$

sum & product $\prod_{j=1}^m \prod_{i=1}^n a_{ij} \geq \prod_{j=1}^m \prod_{i=1}^n a_{i\pi(j)}$ and $\prod_{j=1}^m \sum_{i=1}^n a_{ij} \leq \prod_{j=1}^m \sum_{i=1}^n a_{i\pi(j)}$
where $0 \leq a_{i1} \leq \dots \leq a_{im}$ for $i = 1, \dots, n$ and π is a permutation of $[n]$.
 $|\prod_{i=1}^n a_i - \prod_{i=1}^n b_i| \leq \sum_{i=1}^n |a_i - b_i|$ for $|a_i|, |b_i| \leq 1$.
 $\prod_{i=1}^n (\alpha + a_i) \geq (1 + \alpha)^n$, where $\prod_{i=1}^n a_i \geq 1$, $a_i > 0$, $\alpha > 0$.

Callebaut $\left(\sum_i a_i^{1+x} b_i^{1-x}\right) \left(\sum_i a_i^{1-x} b_i^{1+x}\right) \geq \left(\sum_i a_i^{1+y} b_i^{1-y}\right) \left(\sum_i a_i^{1-y} b_i^{1+y}\right)$
for $1 \geq x \geq y \geq 0$, and $i = 1, \dots, n$.

Karamata $\sum_{i=1}^n \varphi(a_i) \geq \sum_{i=1}^n \varphi(b_i)$ for $a_1 \geq a_2 \geq \dots \geq a_n$ and $b_1 \geq \dots \geq b_n$,
and $\{a_i\} \succeq \{b_i\}$ (majorization), i.e. $\sum_{i=1}^t a_i \geq \sum_{i=1}^t b_i$ for all $1 \leq t \leq n$,
with equality for $t = n$ and φ is convex (for concave φ the reverse holds).

Muirhead $\frac{1}{n!} \sum_{\pi} x_{\pi(1)}^{a_1} \dots x_{\pi(n)}^{a_n} \geq \frac{1}{n!} \sum_{\pi} x_{\pi(1)}^{b_1} \dots x_{\pi(n)}^{b_n}$
where $a_1 \geq a_2 \geq \dots \geq a_n$ and $b_1 \geq b_2 \geq \dots \geq b_n$ and $\{a_k\} \succeq \{b_k\}$,
 $x_i \geq 0$ and the sums extend over all permutations π of $[n]$.

Hilbert $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_m b_n}{m+n} \leq \pi \left(\sum_{m=1}^{\infty} a_m^2\right)^{\frac{1}{2}} \left(\sum_{n=1}^{\infty} b_n^2\right)^{\frac{1}{2}}$ for $a_m, b_n \in \mathbb{R}$.
With $\max\{m, n\}$ instead of $m+n$, we have 4 instead of π .

Hardy $\sum_{n=1}^{\infty} \left(\frac{a_1+a_2+\dots+a_n}{n}\right)^p \leq \left(\frac{p}{p-1}\right)^p \sum_{n=1}^{\infty} a_n^p$ for $a_n \geq 0$, $p > 1$.

Carlson $(\sum_{n=1}^{\infty} a_n)^4 \leq \pi^2 \sum_{n=1}^{\infty} a_n^2 \sum_{n=1}^{\infty} n^2 a_n^2$ for $a_n \in \mathbb{R}$.

Mathieu $\frac{1}{c^2+1/2} < \sum_{n=1}^{\infty} \frac{2n}{(n^2+c^2)^2} < \frac{1}{c^2}$ for $c \neq 0$.

LYM $\sum_{X \in \mathcal{A}} \binom{n}{|X|}^{-1} \leq 1$, $\mathcal{A} \subset 2^{[n]}$, no set in \mathcal{A} is subset of another set in \mathcal{A} .

Sauer-Shelah $|\mathcal{A}| \leq |\text{str}(\mathcal{A})| \leq \sum_{i=0}^{\text{vc}(\mathcal{A})} \binom{n}{i}$ for $\mathcal{A} \subseteq 2^{[n]}$, and
 $\text{str}(\mathcal{A}) = \{X \subseteq [n] : X \text{ shattered by } \mathcal{A}\}$, $\text{vc}(\mathcal{A}) = \max\{|X| : X \in \text{str}(\mathcal{A})\}$.

Bonferroni $\Pr\left[\bigvee_{i=1}^k A_i\right] \leq \sum_{j=1}^k (-1)^{j-1} S_j$ for $1 \leq k \leq n$, k odd,
 $\Pr\left[\bigvee_{i=1}^k A_i\right] \geq \sum_{j=1}^k (-1)^{j-1} S_j$ for $2 \leq k \leq n$, k even.

$S_k = \sum_{1 \leq i_1 < \dots < i_k \leq n} \Pr[A_{i_1} \wedge \dots \wedge A_{i_k}]$ where A_i are events.

Bhatia-Davis $\text{Var}[X] \leq (M - \mathbb{E}[X])(\mathbb{E}[X] - m)$ where $X \in [m, M]$.

Samuelson $\mu - \sigma\sqrt{n-1} \leq x_i \leq \mu + \sigma\sqrt{n-1}$ for $i = 1, \dots, n$.
Where $\mu = \sum x_i/n$, $\sigma^2 = \sum (x_i - \mu)^2/n$.

Markov	$\Pr[X \geq a] \leq E[X]/a$ where X is a random variable (r.v.), $a > 0$. $\Pr[X \leq c] \leq (1 - E[X])/(1 - c)$ for $X \in [0, 1]$ and $c \in [0, E[X]]$. $\Pr[X \in S] \leq E[f(X)]/s$ for $f \geq 0$, and $f(x) \geq s > 0$ for all $x \in S$.	Paley-Zygmund	$\Pr[X \geq \mu E[X]] \geq 1 - \frac{\text{Var}[X]}{(1 - \mu)^2 (E[X])^2 + \text{Var}[X]}$ for $X \geq 0$, $\text{Var}[X] < \infty$, and $\mu \in (0, 1)$.
Chebyshev	$\Pr[X - E[X] \geq t] \leq \text{Var}[X]/t^2$ where $t > 0$. $\Pr[X - E[X] \geq t] \leq \text{Var}[X]/(\text{Var}[X] + t^2)$ where $t > 0$.	Vysochanskij-Petunin-Gauss	$\Pr[X - E[X] \geq \lambda\sigma] \leq \frac{4}{9\lambda^2}$ if $\lambda \geq \sqrt{\frac{8}{3}}$, $\Pr[X - m \geq \varepsilon] \leq \frac{4\tau^2}{9\varepsilon^2}$ if $\varepsilon \geq \frac{2\tau}{\sqrt{3}}$, $\Pr[X - m \geq \varepsilon] \leq 1 - \frac{\varepsilon}{\sqrt{3}\tau}$ if $\varepsilon \leq \frac{2\tau}{\sqrt{3}}$. Where X is a unimodal r.v. with mode m , $\sigma^2 = \text{Var}[X] < \infty$, $\tau^2 = \text{Var}[X] + (E[X] - m)^2 = E[(X - m)^2]$.
2nd moment	$\Pr[X > 0] \geq (E[X])^2/(E[X^2])$ where $E[X] \geq 0$. $\Pr[X = 0] \leq \text{Var}[X]/(E[X^2])$ where $E[X^2] \neq 0$.	Etemadi	$\Pr[\max_{1 \leq k \leq n} S_k \geq 3\alpha] \leq 3 \max_{1 \leq k \leq n} (\Pr[S_k \geq \alpha])$ where X_i are i.r.v., $S_k = \sum_{i=1}^k X_i$, $\alpha \geq 0$.
kth moment	$\Pr[X - \mu \geq t] \leq \frac{E[(X - \mu)^k]}{t^k}$ and $\Pr[X - \mu \geq t] \leq C_k \left(\frac{nk}{t^2}\right)^{k/2}$ for $X_i \in [0, 1]$ k -wise indep. r.v., $X = \sum X_i$, $i = 1, \dots, n$, $\mu = E[X]$, $C_k = 2\sqrt{\pi k} e^{1/6k} \leq 1.0004$, k even.	Doob	$\Pr[\max_{1 \leq k \leq n} X_k \geq \varepsilon] \leq E[X_n]/\varepsilon$ for martingale (X_k) and $\varepsilon > 0$.
2nd and 4th	$E[X] \geq \frac{(E[X^2])^{3/2}}{(E[X^4])^{1/2}}$ where $0 < E[X^4] < \infty$. $\Pr[X \geq \frac{\sigma}{2\sqrt{t}}] > 0$ where $E[X] = 0$, $E[X^2] = \sigma^2$, $0 < E[X^4] \leq t\sigma^4$.	Bennett	$\Pr[\sum_{i=1}^n X_i \geq \varepsilon] \leq \exp\left(-\frac{n\sigma^2}{M^2} \theta\left(\frac{M\varepsilon}{n\sigma^2}\right)\right)$ where X_i i.r.v., $E[X_i] = 0$, $\sigma^2 = \frac{1}{n} \sum \text{Var}[X_i]$, $ X_i \leq M$ (w. prob. 1), $\varepsilon \geq 0$, $\theta(u) = (1 + u) \log(1 + u) - u$.
Chernoff	$\Pr[X \geq t] \leq F(a)/a^t$ for X r.v., $\Pr[X = k] = p_k$, $F(z) = \sum_k p_k z^k$ probability gen. func., and $a \geq 1$. $\Pr[X \geq (1 + \delta)\mu] \leq \left(\frac{e^\delta}{(1 + \delta)(1 + \delta)}\right)^\mu \leq \exp\left(\frac{-\mu\delta^2}{3}\right)$ for X_i i.r.v. from $[0, 1]$, $X = \sum X_i$, $\mu = E[X]$, $\delta \geq 0$ resp. $\delta \in [0, 1]$. $\Pr[X \leq (1 - \delta)\mu] \leq \left(\frac{e^{-\delta}}{(1 - \delta)(1 - \delta)}\right)^\mu \leq \exp\left(\frac{-\mu\delta^2}{2}\right)$ for $\delta \in [0, 1]$. Further from the mean: $\Pr[X \geq R] \leq 2^{-R}$ for $R \geq 2e\mu$ ($\approx 5.44\mu$). $\Pr[X \geq t] \leq \frac{\binom{n}{k} p^k}{\binom{n}{t}}$ for $X_i \in \{0, 1\}$ k -wise i.r.v., $E[X_i] = p$, $X = \sum X_i$. $\Pr[X \geq (1 + \delta)\mu] \leq \frac{\binom{n}{k} p^k}{\binom{n}{(1 + \delta)\mu}} / \binom{n}{k}^{(1 + \delta)\mu}$ for $X_i \in [0, 1]$ k -wise i.r.v., $k \geq \hat{k} = \lceil \mu\delta/(1 - p) \rceil$, $E[X_i] = p_i$, $X = \sum X_i$, $\mu = E[X]$, $p = \frac{\mu}{n}$, $\delta > 0$.	Bernstein	$\Pr[\sum_{i=1}^n X_i \geq \varepsilon] \leq \exp\left(\frac{-\varepsilon^2}{2(n\sigma^2 + M\varepsilon/3)}\right)$ for X_i i.r.v., $E[X_i] = 0$, $ X_i < M$ (w. prob. 1) for all i , $\sigma^2 = \frac{1}{n} \sum \text{Var}[X_i]$, $\varepsilon \geq 0$.
		Azuma	$\Pr[X_n - X_0 \geq \delta] \leq 2 \exp\left(\frac{-\delta^2}{2 \sum_{i=1}^n c_i^2}\right)$ for martingale (X_k) s.t. $ X_i - X_{i-1} < c_i$ (w. prob. 1), for $i = 1, \dots, n$, $\delta \geq 0$.
		Efron-Stein	$\text{Var}[Z] \leq \frac{1}{2} E\left[\sum_{i=1}^n (Z - Z^{(i)})^2\right]$ for $X_i, X_i' \in \mathcal{X}$ i.r.v., $f: \mathcal{X}^n \rightarrow \mathbb{R}$, $Z = f(X_1, \dots, X_n)$, $Z^{(i)} = f(X_1, \dots, X_i', \dots, X_n)$.
		McDiarmid	$\Pr[Z - E[Z] \geq \delta] \leq 2 \exp\left(\frac{-2\delta^2}{\sum_{i=1}^n c_i^2}\right)$ for $X_i, X_i' \in \mathcal{X}$ i.r.v., $Z, Z^{(i)}$ as before, s.t. $ Z - Z^{(i)} \leq c_i$ for all i , and $\delta \geq 0$.
		Janson	$M \leq \Pr[\bigwedge \bar{B}_i] \leq M \exp\left(\frac{\Delta}{2 - 2\varepsilon}\right)$ where $\Pr[B_i] \leq \varepsilon$ for all i , $M = \prod (1 - \Pr[B_i])$, $\Delta = \sum_{i \neq j, B_i \sim B_j} \Pr[B_i \wedge B_j]$.
		Lovász	$\Pr[\bigwedge \bar{B}_i] \geq \prod (1 - x_i) > 0$ where $\Pr[B_i] \leq x_i \cdot \prod_{(i,j) \in D} (1 - x_j)$, for $x_i \in [0, 1]$ for all $i = 1, \dots, n$ and D the dependency graph. If each B_i mutually indep. of all other events, exc. at most d , $\Pr[B_i] \leq p$ for all $i = 1, \dots, n$, then if $ep(d + 1) \leq 1$ then $\Pr[\bigwedge \bar{B}_i] > 0$.
Kolmogorov	$\Pr[\max_k S_k \geq \varepsilon] \leq \frac{1}{\varepsilon^2} \text{Var}[S_n] = \frac{1}{\varepsilon^2} \sum_i \text{Var}[X_i]$ where X_1, \dots, X_n are i.r.v., $E[X_i] = 0$, $\text{Var}[X_i] < \infty$ for all i , $S_k = \sum_{i=1}^k X_i$ and $\varepsilon > 0$.		