Useful Inequalities $\{x^2 \geqslant 0\}$

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Cauchy-Schwarz $$\left(\sum_{i=1}^n x_i y_i\right)^2 \le \left(\sum_{i=1}^n x_i^2\right)\left(\sum_{i=1}^n y_i^2\right)$$

Minkowski $$\left(\sum_{i=1}^n |x_i+y_i|^p\right)^{\frac{1}{p}} \le \left(\sum_{i=1}^n |x_i|^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^n |y_i|^p\right)^{\frac{1}{p}} \quad \text{for } p \ge 1.$$

Hölder $$\sum_{i=1}^n |x_i y_i| \le \left(\sum_{i=1}^n |x_i|^p\right)^{1/p} \left(\sum_{i=1}^n |y_i|^q\right)^{1/q} \quad \text{for } p,q>1,\ \frac{1}{p}+\frac{1}{q}=1.$$

Bernoulli

$(1+x)^r \ge 1+rx$ for $x > -1$, $r \in \mathbb{R} \setminus (0,1)$. Reverse for $r \in [0,1]$.

$(1+x)^{2n} \ge 1+2nx$ for $x \in \mathbb{R}$, $n \in \mathbb{N}$.

$(1+x)^r \le 1 + \dfrac{rx}{1-(r-1)x}$ for $x \in \left[-1, \dfrac{1}{r-1}\right)$, $r > 1$.

exponential

$e^x \ge \left(1+\frac{x}{n}\right)^n \ge 1+x$, $\quad \left(1+\frac{x}{n}\right)^n \ge e^x\left(1-\frac{x^2}{n}\right)$ for $n>1$, $|x| \le n$.

$e^x \ge x^e$ for $x \in \mathbb{R}$, $\quad e^x \ge 1+x+\dfrac{x^2}{2}$ for $x \ge 0$, reverse for $x \le 0$.

$e^x \le \left(1+\frac{x}{n}\right)^{n+x/2}$ for $x, n > 0$.

logarithm

$\dfrac{x}{x+1} \le \ln(1+x) \le \min\left\{x,\ x - \dfrac{x^2}{2} + \dfrac{x^3}{3}\right\}$ for $x > -1$.

$\dfrac{2x}{2+x} \le \ln(1+x) \le \dfrac{x}{\sqrt{x+1}}$ for $x \ge 0$. Reverse for $x \in (-1, 0]$.

$\ln(1+x) \ge x - \dfrac{x^2}{2} + \dfrac{x^3}{4}$ for $x \in [0, \sim 0.45]$, reverse elsewhere.

$\ln(1-x) \ge -x - \dfrac{x^2}{2} - \dfrac{x^3}{2}$ for $x \in [0, \sim 0.43]$, reverse elsewhere.

harmonic $$\ln(n+1) \le \sum_{i=1}^n \frac{1}{i} \le \ln(n)+1$$

square root $$2\sqrt{x+1} - 2\sqrt{x} < \frac{1}{\sqrt{x}} < 2\sqrt{x} - 2\sqrt{x-1} \quad \text{for } x \ge 1.$$

binomial $$\left(\frac{n}{k}\right)^k \le \binom{n}{k} \le \frac{n^k}{k!} \le \left(\frac{en}{k}\right)^k \quad \text{and} \quad \binom{n}{k} < 2^n \quad \text{for } n \ge k > 0.$$

binomial sum

$$\sum_{i=0}^d \binom{n}{i} \le n^d + 1 \quad \text{and} \quad \sum_{i=0}^d \binom{n}{i} \le 2^n \quad \text{for } n \ge d \ge 0,$$

$$\sum_{i=0}^d \binom{n}{i} \le \left(\frac{en}{d}\right)^d \quad \text{for } n \ge d \ge 1.$$

binomial ratio $$\frac{2^{2n}}{2\sqrt{n}} \le \binom{2n}{n} \le \frac{2^{2n}}{\sqrt{2n}} \quad \text{and} \quad \binom{n}{\alpha n} \le \left[\alpha^\alpha (1-\alpha)^{(1-\alpha)}\right]^{-n} \quad \text{for } \alpha \in (0,1).$$

Stirling $$e\left(\frac{n}{e}\right)^n \le \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/(12n+1)} \le n! \le \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/12n} \le en\left(\frac{n}{e}\right)^n$$

trigonometric hyperbolic

$x - \dfrac{x^3}{2} \le x\cos x \le \dfrac{x \cos x}{1-x^2/3} \le x\sqrt[3]{\cos x} \le x - x^3/6 \le x \cos\dfrac{x}{\sqrt{3}} \le \sin x,$

$x \cos x \le \dfrac{x^3}{\sinh^2 x} \le x\cos^2(x/2) \le \sin x \le (x\cos x + 2x)/3 \le \dfrac{x^2}{\sinh x},$

$\dfrac{2}{\pi}x \le \sin x \le x\cos(x/2) \le x \le x + x^3/3 \le \tan x$ all for $x \in \left[0, \frac{\pi}{2}\right]$.

$\cosh(x) + \alpha\sinh(x) \le e^{x(\alpha+x/2)}$ for $x \in \mathbb{R}$, $\alpha \in [-1,1]$.

Napier $$b > \frac{a+b}{2} > \frac{b-a}{\ln(b)-\ln(a)} > \sqrt{ab} > a \quad \text{where } 0<a<b.$$

means $$\max\{x_i\} \ge \sqrt{\frac{\sum {x_i}^2}{n}} \ge \frac{\sum x_i}{n} \ge \left(\prod x_i\right)^{1/n} \ge \frac{n}{\sum x_i^{-1}} \ge \min\{x_i\}$$

power means

$M_w^r \le M_w^s$ for all pairs $r \le s$, where:

$$M_w^r(x_1, x_2, \ldots, x_n) = \left(\sum_{i=1}^n w_i x_i^r\right)^{1/r} \quad \text{and} \quad \sum w_i = 1.$$

If $r = -\infty, 0, +\infty$, M_w^r tends to min, geom. mean and max, respectively.

Maclaurin

$\sqrt[k]{S_k} \ge \sqrt[(k+1)]{S_{k+1}}$ for $1 \le k < n$, where:

$$S_k = \frac{1}{\binom{n}{k}} \sum_{1 \le i_1 < \cdots < i_k \le n} a_{i_1} a_{i_2} \cdots a_{i_k}, \quad \text{and} \quad a_i > 0.$$

Newton $${S_k}^2 \ge S_{k-1}S_{k+1} \quad \text{for } 1 \le k < n, \text{ and } S_k \text{ as before.}$$

Jensen $$\varphi\left(\sum_{i=1}^n p_i x_i\right) \le \sum_{i=1}^n p_i \varphi(x_i) \quad \text{where } p_i \ge 0,\ \sum p_i = 1, \text{ and } \varphi \textbf{ convex}.$$

With expectations: $\varphi(\mathrm{E}[X]) \le \mathrm{E}[\varphi(X)]$. For concave φ the reverse holds.

Chebyshev $$\sum_{i=1}^n f(a_i)g(b_i)p_i \ge \left(\sum_{i=1}^n f(a_i)p_i\right)\left(\sum_{i=1}^n g(b_i)p_i\right) \ge \sum_{i=1}^n f(a_i)g(b_{n-i+1})p_i$$

for $a_1 \le \cdots \le a_n$, $b_1 \le \cdots \le b_n$ and f, g nondecreasing, $p_i \ge 0$, $\sum p_i = 1$.

With expectations: $\mathrm{E}\big[f(X)g(X)\big] \ge \mathrm{E}\big[f(X)\big]\mathrm{E}\big[g(X)\big]$.

rearrangement $$\sum_{i=1}^n a_i b_i \ge \sum_{i=1}^n a_i b_{\pi(i)} \ge \sum_{i=1}^n a_i b_{n-i+1} \quad \text{for } a_1 \le \cdots \le a_n,$$

$b_1 \le \cdots \le b_n$ and π a permutation of $[n]$. More generally:

$$\sum_{i=1}^n f_i(b_i) \ge \sum_{i=1}^n f_i(b_{\pi(i)}) \ge \sum_{i=1}^n f_i(b_{n-i+1})$$

with $\big(f_{i+1}(x) - f_i(x)\big)$ nondecreasing for all $1 \le i < n$.

Young $$xy \le \frac{x^p}{p} + \frac{y^q}{q} \quad \text{for } x, y \ge 0,\ \ p, q > 0 \ \text{ and } \ \frac{1}{p}+\frac{1}{q} = 1.$$

Chong $$\sum_{i=1}^n \frac{a_i}{a_{\pi(i)}} \ge n \quad \text{and} \quad \prod_{i=1}^n {a_i}^{a_i} \ge \prod_{i=1}^n {a_i}^{a_{\pi(i)}} \quad \text{for } a_i > 0.$$

Kantorovich $\left(\sum_{i=1}^{n} {x_i}^2\right)\left(\sum_{i=1}^{n} {y_i}^2\right) \le \left(\frac{A}{G}\right)^2 \left(\sum_{i=1}^{n} x_i y_i\right)^2$ for $x_i, y_i > 0$,
$0 < m \le \frac{x_i}{y_i} \le M < \infty$, $A = (m+M)/2$, $G = \sqrt{mM}$.

Cauchy $\varphi'(a) \le \frac{f(b)-f(a)}{b-a} \le \varphi'(b)$ where $a < b$, and φ convex.

Hermite $\varphi\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b \varphi(x)\,dx \le \frac{\varphi(a)+\varphi(b)}{2}$ for φ convex.

Gibbs $\sum_{i=1}^{n} a_i \log\frac{a_i}{b_i} \ge a \log\frac{a}{b}$ for $a_i, b_i \ge 0$, $a := \sum a_i$, $b := \sum b_i$.

Woeginger $\sum_{i=1}^{n} a_i \varphi(\frac{b_i}{a_i}) \le a\,\varphi(\frac{b}{a})$ for φ **concave** and variables as before.

Pečarić $\left(1+\frac{x}{p}\right)^p \ge \left(1+\frac{x}{q}\right)^q$ where either (i) $x > 0$, $p > q > 0$,
(ii) $-p < -q < x < 0$ or (iii) $-q > -p > x > 0$. Reverse, if
(iv) $q < 0 < p$, $-q > x > 0$ or (v) $q < 0 < p$, $-p < x < 0$.

Shapiro $\sum_{i=1}^{n} \frac{x_i}{x_{i+1}+x_{i+2}} \ge \frac{n}{2}$ where $x_i > 0$, $(x_{n+1}, x_{n+2}) := (x_1, x_2)$,
and $n \le 12$ if even, $n \le 23$ if odd.

Schur $x^t(x-y)(x-z) + y^t(y-z)(y-x) + z^t(z-x)(z-y) \ge 0$
where $x, y, z \ge 0$, $t > 0$

Weierstrass $\prod_{i=1}^{n} (1-x_i)^{w_i} \ge 1 - \sum_{i=1}^{n} w_i x_i$ where $x_i \le 1$ and
either $w_i \ge 1$ (for all i) or $w_i \le 0$ (for all i).
If $w_i \in [0,1]$, $\sum w_i \le 1$ and $x_i \le 1$, the reverse holds.

Hadamard $(\det A)^2 \le \prod_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2$ where A is an $n \times n$ matrix.

Ky Fan $\frac{\prod_{i=1}^{n} x_i^{a_i}}{\prod_{i=1}^{n} (1-x_i)^{a_i}} \le \frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} a_i(1-x_i)}$ for $x_i \in [0, \frac{1}{2}]$, $a_i \in [0,1]$, $\sum a_i = 1$.

Aczél $\left(a_1 b_1 - \sum_{i=2}^{n} a_i b_i\right)^2 \ge \left(a_1^2 - \sum_{i=2}^{n} a_i^2\right)\left(b_1^2 - \sum_{i=2}^{n} b_i^2\right)$
given that $a_1^2 > \sum_{i=2}^{n} a_i^2$ or $b_1^2 > \sum_{i=2}^{n} b_i^2$.

Mahler $\prod_{i=1}^{n} (x_i + y_i)^{1/n} \ge \prod_{i=1}^{n} x_i^{1/n} + \prod_{i=1}^{n} y_i^{1/n}$ where $x_i, y_i > 0$.

Callebaut $\left(\sum_{i=1}^{n} a_i^{1+x} b_i^{1-x}\right)\left(\sum_{i=1}^{n} a_i^{1-x} b_i^{1+x}\right) \ge \left(\sum_{i=1}^{n} a_i^{1+y} b_i^{1-y}\right)\left(\sum_{i=1}^{n} a_i^{1-y} b_i^{1+y}\right)$
for $1 \ge x \ge y \ge 0$.

unknown $\sum_{j=1}^{m}\prod_{i=1}^{n} a_{ij} \ge \sum_{j=1}^{m}\prod_{i=1}^{n} a_{i\pi(j)}$ and $\prod_{j=1}^{m}\sum_{i=1}^{n} a_{ij} \le \prod_{j=1}^{m}\sum_{i=1}^{n} a_{i\pi(j)}$
where $0 \le a_{i1} \le \cdots \le a_{im}$ for $i = 1, \ldots, n$ and π is a permutation of $[n]$.

Karamata $\sum_{i=1}^{n} \varphi(a_i) \ge \sum_{i=1}^{n} \varphi(b_i)$ where $a_1 \ge a_2 \ge \cdots \ge a_n$ and $b_1 \ge \cdots \ge b_n$,
and $\{a_i\} \succeq \{b_i\}$ (majorization), i.e. $\sum_{i=1}^{t} a_i \ge \sum_{i=1}^{t} b_i$ for all $1 \le t \le n$,
with equality for $t = n$ and φ is convex (for concave φ the reverse holds).

Muirhead $\frac{1}{n!}\sum_{\pi} x_{\pi(1)}^{a_1}\cdots x_{\pi(n)}^{a_n} \ge \frac{1}{n!}\sum_{\pi} x_{\pi(1)}^{b_1}\cdots x_{\pi(n)}^{b_n}$
where $a_1 \ge a_2 \ge \cdots \ge a_n$ and $b_1 \ge b_2 \ge \cdots \ge b_n$ and $\{a_k\} \succeq \{b_k\}$,
$x_i \ge 0$ and the sums extend over all permutations π of $[n]$.

Carleman $\sum_{k=1}^{n}\left(\prod_{i=1}^{k} |a_i|\right)^{1/k} \le e \sum_{k=1}^{n} |a_k|$

Milne $\left(\sum_{i=1}^{n}(a_i + b_i)\right)\left(\sum_{i=1}^{n}\frac{a_i b_i}{a_i + b_i}\right) \le \left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} b_i\right)$

Abel $b_n \min_k \sum_{i=1}^{k} |a_i| \le \sum_{i=1}^{n} |a_i b_i| \le b_n \max_k \sum_{i=1}^{k} |a_i|$ for $0 \le b_1 \le \cdots \le b_n$.

Hilbert $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_m b_n}{m+n} \le \pi\left(\sum_{m=1}^{\infty} a_m^2\right)^{\frac{1}{2}}\left(\sum_{n=1}^{\infty} b_n^2\right)^{\frac{1}{2}}$ for $a_m, b_n \in \mathbb{R}$.
If we put $\max\{m, n\}$ instead of $m+n$, we have 4 instead of π.

Hardy $\sum_{n=1}^{\infty}\left(\frac{a_1 + a_2 + \cdots + a_n}{n}\right)^p \le \left(\frac{p}{p-1}\right)^p \sum_{n=1}^{\infty} a_n^p$ for $a_n \ge 0$, $p > 1$.

Carlson $\left(\sum_{n=1}^{\infty} a_n\right)^4 \le \pi^2 \sum_{n=1}^{\infty} {a_n}^2 \sum_{n=1}^{\infty} n^2 {a_n}^2$ for $a_n \in \mathbb{R}$.

Mathieu $\frac{1}{c^2 + 1/2} < \sum_{n=1}^{\infty}\frac{2n}{(n^2+c^2)^2} < \frac{1}{c^2}$ for $c \ne 0$.

Copson $\sum_{n=1}^{\infty}\left(\sum_{k \ge n}\frac{a_k}{k}\right)^p \le p^p \sum_{n=1}^{\infty} {a_n}^p$ for $a_n \ge 0$, $p > 1$, reverse if $p \in (0,1)$.

Bhatia-Davis $\mathrm{Var}[X] \le \left(M - \mathrm{E}[X]\right)\left(\mathrm{E}[X] - m\right)$ where $X \in [m, M]$.

Bonferroni

$$\Pr\Big[\bigvee_{i=1}^{n} A_i\Big] \le \sum_{j=1}^{k}(-1)^{j-1}S_j \quad \text{for } 1 \le k \le n,\ k \text{ odd},$$

$$\Pr\Big[\bigvee_{i=1}^{n} A_i\Big] \ge \sum_{j=1}^{k}(-1)^{j-1}S_j \quad \text{for } 2 \le k \le n,\ k \text{ even}.$$

$$S_k = \sum_{1\le i_1<\cdots<i_k\le n} \Pr\big[A_{i_1}\wedge\cdots\wedge A_{i_k}\big] \quad \text{where } A_i \text{ are events.}$$

Markov

$$\Pr\big[|X| \ge a\,\big] \le \frac{\mathrm{E}\big[|X|\big]}{a} \quad \text{where } X \text{ is a random variable, } a > 0.$$

$$\Pr\big[X \le c\,\big] \le \frac{1-\mathrm{E}[X]}{1-c} \quad \text{for } X\in[0,1] \text{ and } c \in \big[0, \mathrm{E}[X]\big].$$

Chebyshev

$$\Pr\Big[\,\big|X-\mathrm{E}[X]\big| \ge t\,\Big] \le \frac{\mathrm{Var}[X]}{t^2}$$

$$\Pr\big[X-\mathrm{E}[X] \ge t\,\big] \le \frac{\mathrm{Var}[X]}{\mathrm{Var}[X]+t^2} \quad \text{where } t>0 \text{ (for both).}$$

Samuelson

$$\mu - \sigma\sqrt{n-1} \le x_i \le \mu + \sigma\sqrt{n-1} \quad \text{for } i = 1,\dots,n.$$

Where $\mu = \sum x_i/n$, $\sigma^2 = \sum (x_i-\mu)^2/n$.

Vysochanskij-Petunin-Gauss

$$\Pr\Big[\,\big|X-\mathrm{E}[X]\big| \ge \lambda\sigma\,\Big] \le \frac{4}{9\lambda^2} \quad \text{if } \lambda \ge \sqrt{\frac{8}{3}},$$

$$\Pr\Big[\,\big|X-m\big| \ge \varepsilon\,\Big] \le \frac{4\tau^2}{9\varepsilon^2} \quad \text{if } \varepsilon \ge \frac{2\tau}{\sqrt{3}},$$

$$\Pr\Big[\,\big|X-m\big| \ge \varepsilon\,\Big] \le 1 - \frac{\varepsilon}{\sqrt{3}\tau} \quad \text{if } \varepsilon \le \frac{2\tau}{\sqrt{3}}.$$

Where X is a unimodal random variable with mode m,
$\sigma^2 = \mathrm{Var}[X] < \infty$, $\tau^2 = \mathrm{Var}[X] + (\mathrm{E}[X]-m)^2 = \mathrm{E}[(X-m)^2]$.

Paley-Zygmund

$$\Pr\big[X \ge \mu\,\mathrm{E}[X]\,\big] \ge 1 - \frac{\mathrm{Var}[X]}{(1-\mu)^2\,(\mathrm{E}[X])^2 + \mathrm{Var}[X]} \quad \text{where } X \ge 0$$

random variable, $\mathrm{Var}[X] < \infty$, and $\mu \in (0,1)$.

Kolmogorov

$$\Pr\big[\max_k |S_k| \ge \varepsilon\big] \le \frac{1}{\varepsilon^2}\mathrm{Var}[S_n] = \frac{1}{\varepsilon^2}\sum_i \mathrm{Var}[X_i]$$

where $X_1,\dots,X_n$ are independent random variables, $\mathrm{E}[X_i]=0$,
$\mathrm{Var}[X_i] < \infty$ for all i, $S_k = \sum_{i=1}^{k} X_i$ and $\varepsilon > 0$.

Etemadi

$$\Pr\big[\max_{1\le k\le n} |S_k| \ge 3\alpha\big] \le 3 \max_{1\le k\le n}\big(\Pr\big[\,|S_k| \ge \alpha\big]\big)$$

where X_i are independent random variables, $S_k = \sum_{i=1}^{k} X_i$, $\alpha \ge 0$.

Doob

$$\Pr\big[\max_{1\le k\le n} |X_k| \ge \varepsilon\big] \le \frac{\mathrm{E}\big[|X_n|\big]}{\varepsilon} \quad \text{for martingale } (X_k) \text{ and } \varepsilon > 0.$$

Bennett

$$\Pr\Big[\sum_{i=1}^{n} X_i \ge \varepsilon\,\Big] \le \exp\left(-\frac{n\sigma^2}{M^2}\,\theta\Big(\frac{M\varepsilon}{n\sigma^2}\Big)\right) \quad \text{where } X_i \text{ independent,}$$

$\mathrm{E}[X_i]=0$, $\sigma^2 = \frac{1}{n}\sum \mathrm{Var}[X_i]$, $|X_i| \le M$ (w. probab. 1), $\varepsilon \ge 0$,
$\theta(u) = (1+u)\log(1+u) - u$.

Bernstein

$$\Pr\Big[\sum_{i=1}^{n} X_i \ge \varepsilon\,\Big] \le \exp\left(\frac{-\varepsilon^2}{2(n\sigma^2 + M\varepsilon/3)}\right) \quad \text{for } X_i \text{ independent,}$$

$\mathrm{E}[X_i]=0$, $|X_i| < M$ (w. prob. 1) for all i, $\sigma^2 = \frac{1}{n}\sum \mathrm{Var}[X_i]$, $\varepsilon \ge 0$.

Chernoff

$$\Pr\big[\,X \ge (1+\delta)\mu\,\big] \le \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu} \le \exp\left(\frac{-\mu\delta^2}{\min\{2+\delta, 3\}}\right)$$

where X_i independently drawn from {0,1}, $X = \sum X_i$, $\mu = \mathrm{E}[X]$, $\delta \ge 0$.

$$\Pr\big[\,X \le (1-\delta)\mu\,\big] \le \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^{\mu} \le \exp\left(\frac{-\mu\delta^2}{2}\right) \quad \text{for } \delta \in [0,1).$$

Simpler (weaker) form: $\Pr\big[X \ge R\,\big] \le 2^{-R}$ for $R \ge 2e\mu$ ($\approx 5.44\mu$).

Hoeffding

$$\Pr\Big[\,\big|X-\mathrm{E}[X]\big| \ge \delta\,\Big] \le 2\exp\left(\frac{-2\delta^2}{\sum_{i=1}^{n}(b_i-a_i)^2}\right) \quad \text{for } X_i \text{ independent,}$$

$X_i \in [a_i, b_i]$ (w. probability 1), $X = \sum X_i$, $\delta \ge 0$.

A related lemma, assuming $\mathrm{E}[X]=0$, $X\in[a,b]$ (w. prob. 1) and $\lambda\in\mathbb{R}$:

$$\mathrm{E}\big[e^{\lambda X}\big] \le \exp\left(\frac{\lambda^2(b-a)^2}{8}\right)$$

Azuma

$$\Pr\Big[\,\big|X_n - X_0\big| \ge \delta\,\Big] \le 2\exp\left(\frac{-\delta^2}{2\sum_{i=1}^{n} {c_i}^2}\right) \quad \text{for martingale } (X_k) \text{ s.t.}$$

$\big|X_i - X_{i-1}\big| < c_i$ (w. prob. 1), for $i = 1,\dots,n$, $\delta \ge 0$.

Efron-Stein

$$\mathrm{Var}[Z] \le \frac{1}{2}\,\mathrm{E}\left[\sum_{i=1}^{n}\big(Z - Z^{(i)}\big)^2\right] \quad \text{for } X_i, {X_i}' \in \mathcal{X} \text{ independent,}$$

$f : \mathcal{X}^n \to \mathbb{R}$, $Z = f(X_1,\dots,X_n)$, $Z^{(i)} = f(X_1,\dots,{X_i}',\dots,X_n)$.

McDiarmid

$$\Pr\Big[\,\big|Z-\mathrm{E}[Z]\big| \ge \delta\,\Big] \le 2\exp\left(\frac{-2\delta^2}{\sum_{i=1}^{n} c_i^2}\right) \quad \text{for } X_i, {X_i}' \in \mathcal{X} \text{ indep.,}$$

Z, $Z^{(i)}$ as before, s.t. $\big|Z - Z^{(i)}\big| \le c_i$ for all i, and $\delta \ge 0$.

Janson

$$M \le \Pr\big[\bigwedge \overline{B}_i\big] \le M\exp\left(\frac{\Delta}{2-2\varepsilon}\right) \quad \text{where } \Pr[B_i] \le \varepsilon \text{ for all } i,$$

$$M = \prod(1-\Pr[B_i]), \quad \Delta = \sum_{i\ne j, B_i\sim B_j} \Pr[B_i \wedge B_j].$$

Lovász

$$\Pr\big[\bigwedge \overline{B}_i\big] \ge \prod(1-x_i) > 0 \quad \text{where } \Pr[B_i] \le x_i \cdot \prod_{(i,j)\in D}(1-x_j),$$

for $x_i \in [0,1)$ for all $i = 1,\dots,n$ and D the dependency graph.
If each B_i mutually indep. of the set of all other events, exc. at most d,
$\Pr[B_i] \le p$ for all $i = 1,\dots,n$, then if $ep(d+1) \le 1$ then $\Pr\big[\bigwedge \overline{B}_i\big] > 0$.

Erdős

$$\sum_{1\le j<k\le n} \frac{1}{x_k - x_j} \ge \frac{1}{8}n^2 \ln n \quad \text{where } -1 \le x_1 \le \cdots \le x_n \le 1.$$

Kraft

$$\sum_{i=1}^{N} 2^{-c(i)} \le 1, \quad N \text{ the \# of leaves in a binary tree, } c \text{ the depth of a leaf.}$$