Part II

Background

Geometry of BST [Demaine, Harmon, Iacono, Kane, Pătrașcu, SODA'09]

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access sequence X
e.g. 4, 5, 6, 1, 2, 3

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$\rightarrow$ point set X

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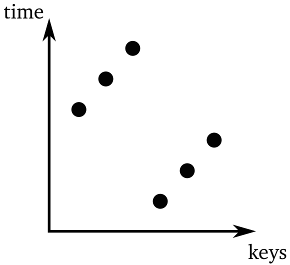

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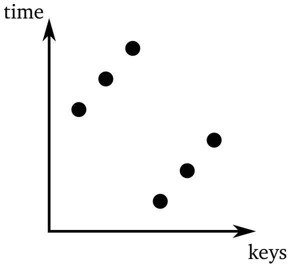
BST algorithm serving X



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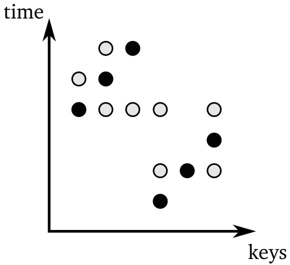
$\rightarrow$ point set $Y \supseteq X$



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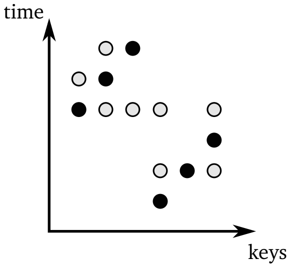
$\rightarrow$ point set $Y \supseteq X$
$\uparrow$
nodes touched by access and rotations at each time



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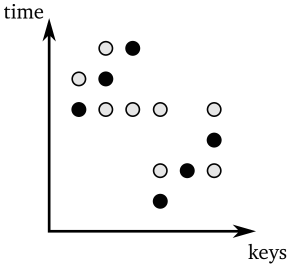

Y is a BST execution of X $\iff$ Y is a satisfied superset of X

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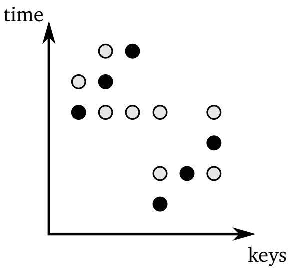

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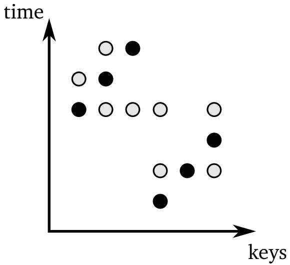

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no $a, b \in Y$ form an empty rectangle

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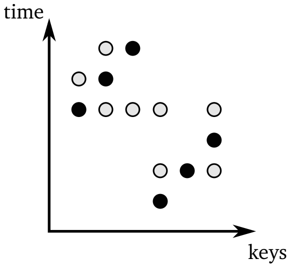

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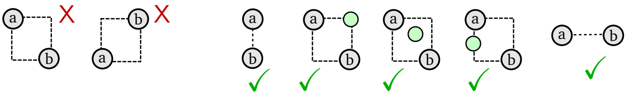

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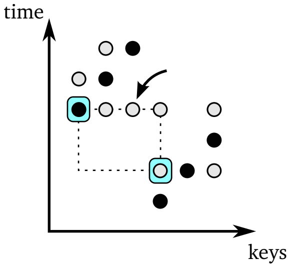

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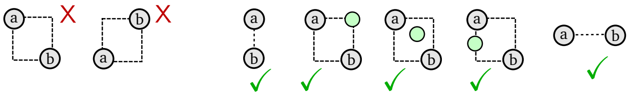

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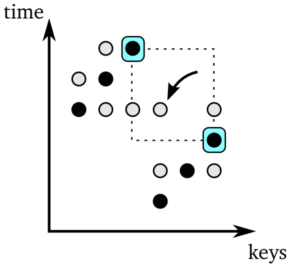

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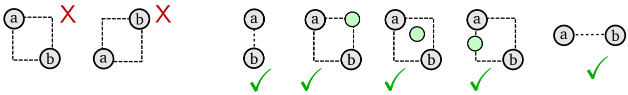

GREEDY [Lucas '88; Munro '00; Demaine et al. '09]

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GREEDY:
a natural offline BST algorithm.

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In geometric view GREEDY
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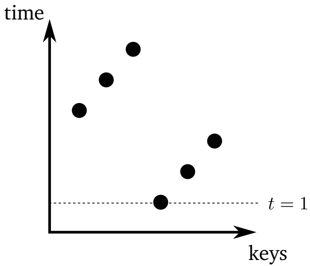

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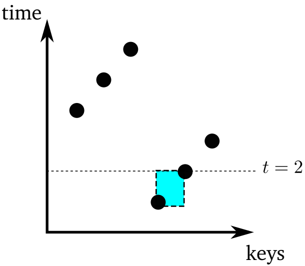

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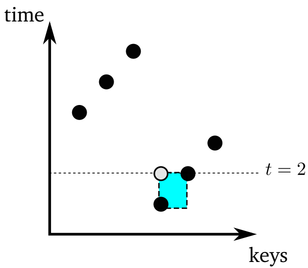

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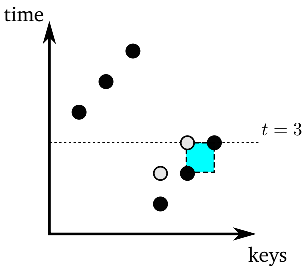

Greedy [Lucas '88; Munro '00; Demaine et al. '09]

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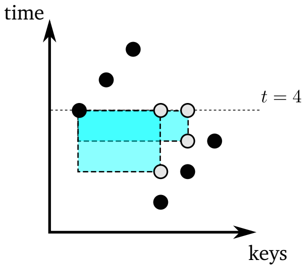

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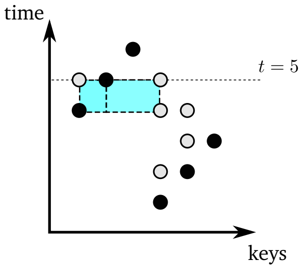

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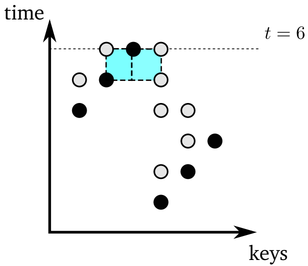

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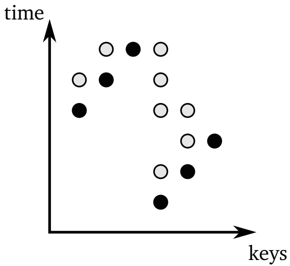

Task: Bound the cost of GREEDY

GREEDY [Lucas '88; Munro '00; Demaine et al. '09]

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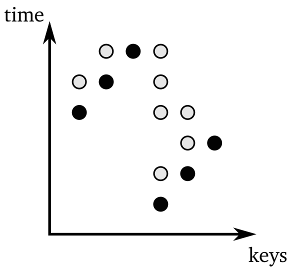

Task: Bound the cost of GREEDY
$\downarrow$
$\approx$ # of points in the GREEDY execution

Forbidden Submatrix Theory

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A useful tool since the 50s.

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Studies patterns in $0/1$-matrices.

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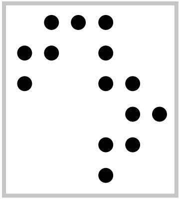
Studies patterns in ~~0/1-matrices~~
points on a grid



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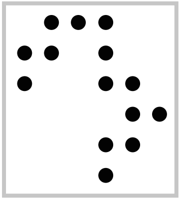

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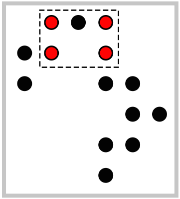

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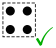

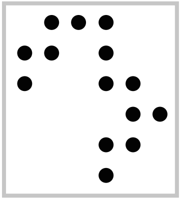

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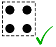

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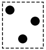

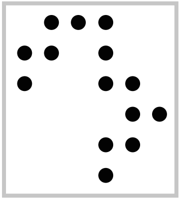

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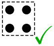

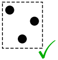

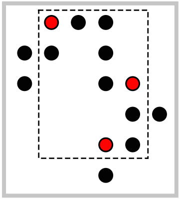

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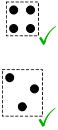

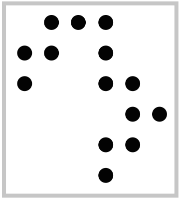

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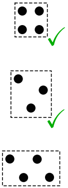

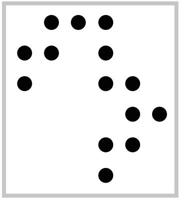

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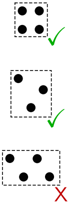

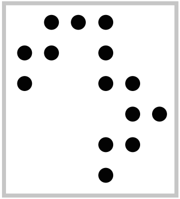

Studies patterns in ~~0/1-matrices~~ points on a grid

How many points can we have while avoiding some pattern?

Forbidden Submatrix Theory

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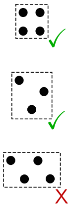

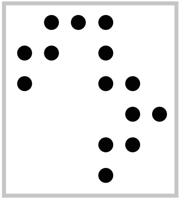

Theorems of the form:

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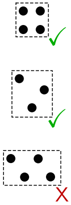
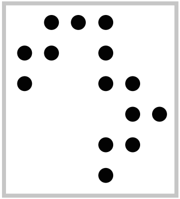
points on a grid





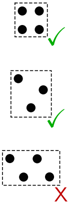
Theorems of the form:

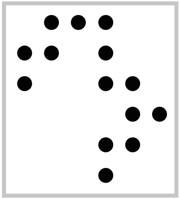
M is a set of points on the n-by-n grid **avoiding** pattern P

Forbidden Submatrix Theory

A useful tool since the 50s.

[Zarankiewicz 1951] [Kővári, Sós, Turán '55] [Bollobás, Erdős '78] [Hart, Sharir '86] [Bienstock, Győri, '91] [Füredi, Hajnal '92] [Marcus, Tardos '04] [Pettie '10]





Studies patterns in ~~0/1-matrices~~
points on a grid

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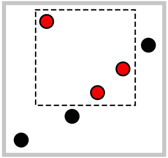
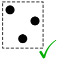
Studies patterns in ~~0/1-matrices~~ points on a grid

Subsumes the pattern-avoidance mentioned earlier:

134562 contains 231





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… back to GREEDY

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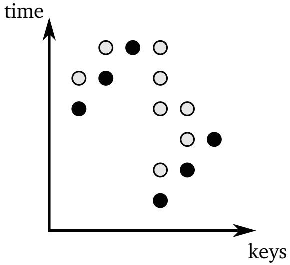
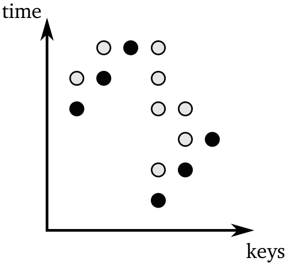


... back to Greedy

We bound the cost of Greedy using forbidden submatrix theory.



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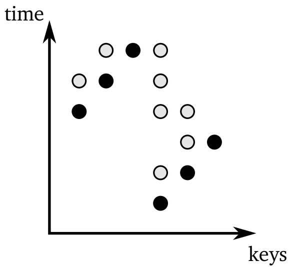
A first (WRONG) conjecture:



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If X avoids P
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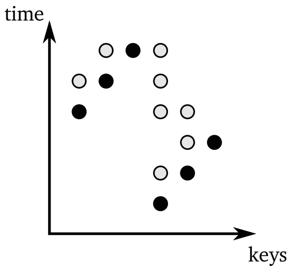

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A first (WRONG) conjecture:

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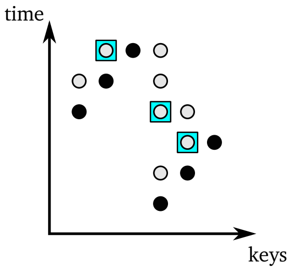

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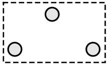
If Greedy execution contains the pattern:

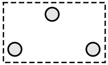


A (correct) **Lemma:**

... back to Greedy

We bound the cost of Greedy using forbidden submatrix theory.

there must be an access point inside



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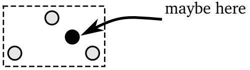

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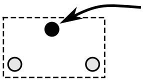

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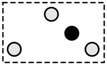

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We call this the **input-revealing** property of GREEDY.

Consequence:

... back to Greedy

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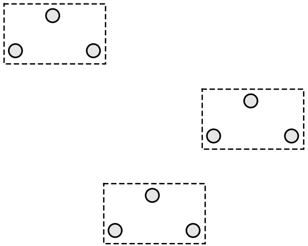

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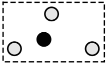

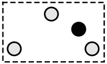

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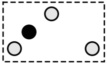

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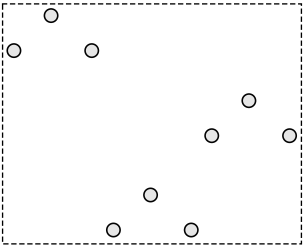

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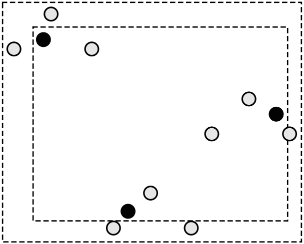

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... back to $\textsc{Greedy}$

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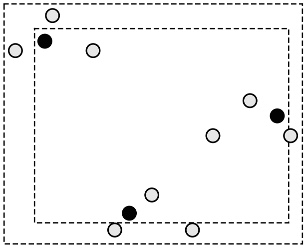

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if X avoids $\begin{pmatrix} 1 & & \\ & & 3 \\ & 2 & \end{pmatrix}$

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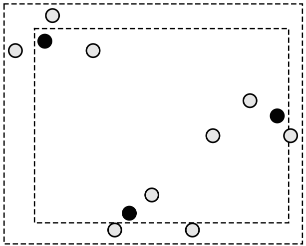

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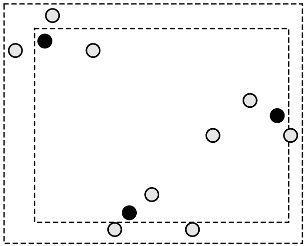

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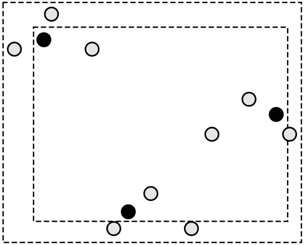
$\implies$ cost of Greedy on X is at most $n \cdot 2^{\mathrm{poly}(\alpha(n))}$

using [Klazar '00] [Keszegh '09] [Pettie '15]

We bound the cost of $\textsc{Greedy}$ using forbidden submatrix theory.



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if X avoids P

$\implies$ $\textsc{Greedy}$ execution avoids $P \otimes (\,.^{\bullet}.\,)$

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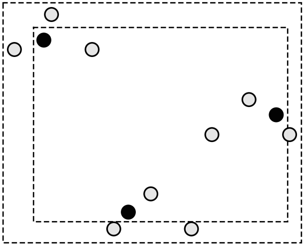

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$\implies$ cost of Greedy on X is $n \cdot 2^{\alpha(n)^{O(|P|)}}$

$\rightarrow$ for various **special cases** we prove stronger bounds, i.e. $O(n)$

proofs more difficult

A different application of the technique...

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Independent Rectangle bound [Demaine et al. '09] [Wilber '89]

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$\rightarrow$ Lower bound on the cost of **any** BST algorithm

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$\rightarrow$ Conjectured to be $\Theta(OPT)$

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We show:
If X **avoids** P, then **IR-bound** for X is $O(n)$, for any constant-sized P.

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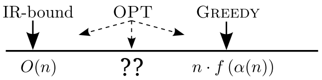

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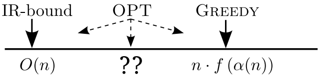

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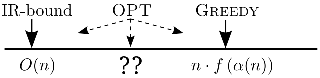

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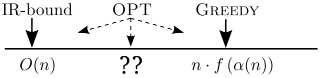

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? GREEDY is in fact linear on all pattern-avoiding input

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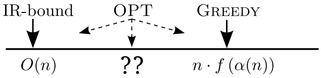

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- ? GREEDY is in fact linear on all pattern-avoiding input
- ? GREEDY is not $O(1)$-competitive

A different application of the technique...

Independent Rectangle bound [Demaine et al. '09] [Wilber '89]

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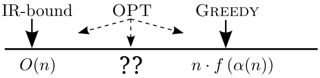

Consequence: "something's gotta give ..." ♪ ♩ ♫

- ? GREEDY is in fact linear on all pattern-avoiding input
- ? GREEDY is not $O(1)$-competitive
- ? Conjecture is false (IR-bound not tight)

Conclusion:

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On inputs that avoid an arbitrary pattern, GREEDY is linear*

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For traversal conjecture, GREEDY is linear*†

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Conclusion:

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Open Question 1
Prove traversal conjecture unconditionally for an online algorithm.

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Open Question 1
Prove traversal conjecture unconditionally for an online algorithm.
Even for preorder sequence of a **path**

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Prove traversal conjecture unconditionally for an online algorithm.
Even for preorder sequence of a **path** $\rightarrow$ sequence avoiding both 231 and 213.

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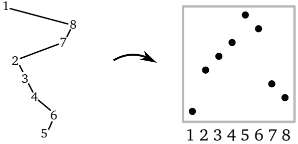

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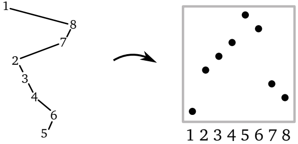

Open Question 2

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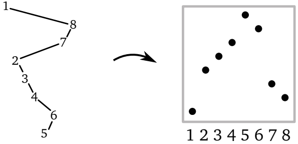

Open Question 2
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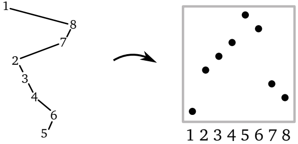

Open Question 2
Prove $o(\log(n))$-competitiveness for GREEDY or Splay Tree, or $o(\log\log(n))$-competitiveness for any algorithm.