Part II

Background

Geometry of BST [Demaine, Harmon, Iacono, Kane, Pătrașcu, SODA'09]

Geometry of BST [Demaine, Harmon, Iacono, Kane, Pătrașcu, SODA'09]

access sequence X
e.g. 4, 5, 6, 1, 2, 3

Geometry of BST [Demaine, Harmon, Iacono, Kane, Pătrașcu, SODA'09]

access sequence X
e.g. 4, 5, 6, 1, 2, 3

$\rightarrow$ point set X

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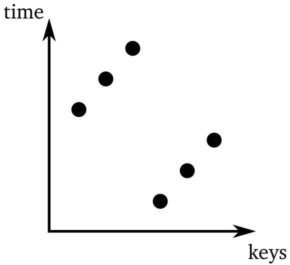

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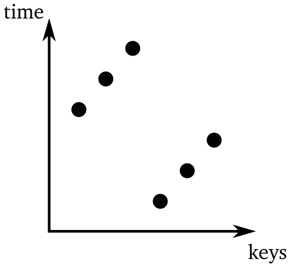
BST algorithm serving X



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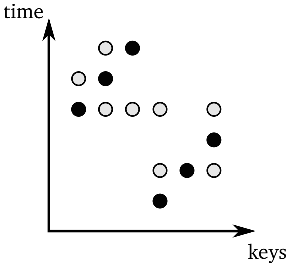
BST algorithm serving X

$\rightarrow$ point set $Y \supseteq X$



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access sequence X
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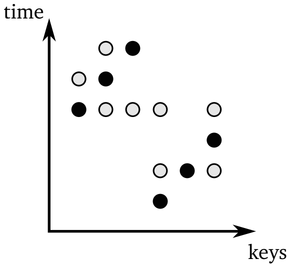
$\rightarrow$ point set X

BST algorithm serving X

$\rightarrow$ point set $Y \supseteq X$
$\uparrow$
nodes touched by access and rotations at each time



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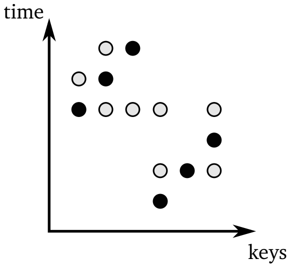

Y is a BST execution of X $\iff$ Y is a satisfied superset of X

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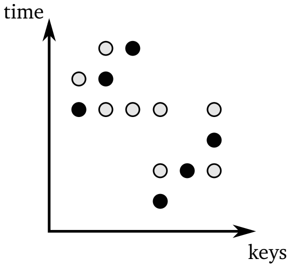

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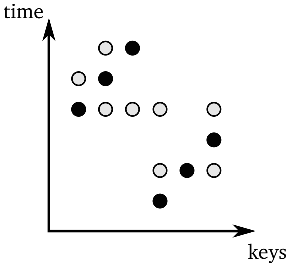

Y is a BST execution of X $\iff$ Y is a satisfied superset of X
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no $a, b \in Y$ form an empty rectangle

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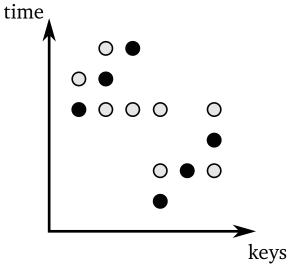

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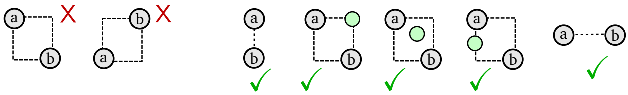

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$\to$ point set X

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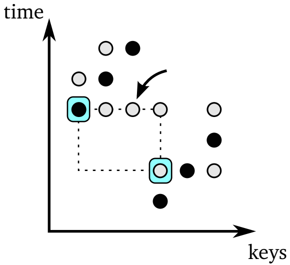

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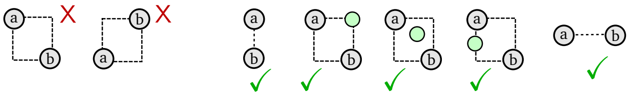

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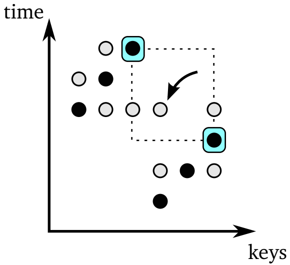

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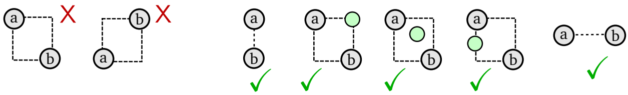

GREEDY [Lucas '88; Munro '00; Demaine et al. '09]

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GREEDY:
a natural offline BST algorithm.

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In geometric view GREEDY
becomes:

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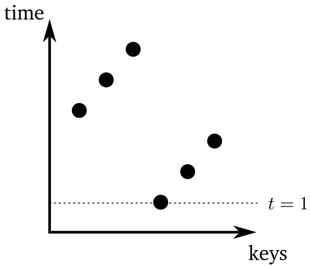

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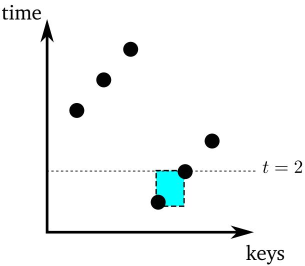

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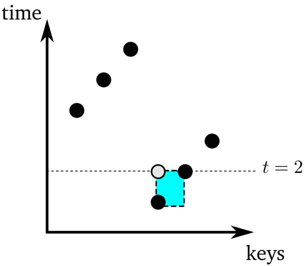

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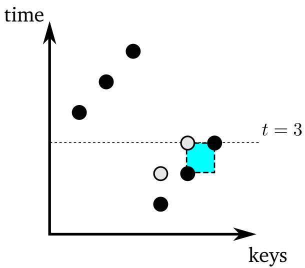

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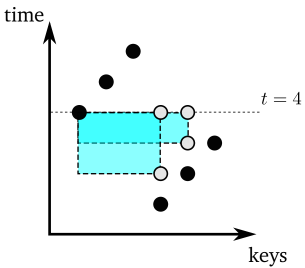

Greedy [Lucas '88; Munro '00; Demaine et al. '09]

Greedy:
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In geometric view Greedy becomes:

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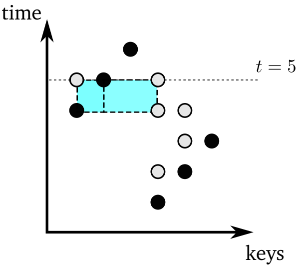

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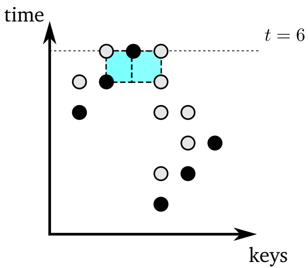

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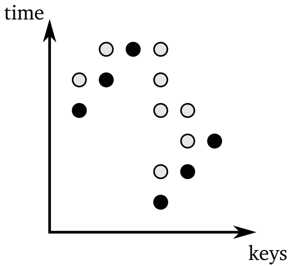

Task: Bound the cost of GREEDY

GREEDY [Lucas '88; Munro '00; Demaine et al. '09]

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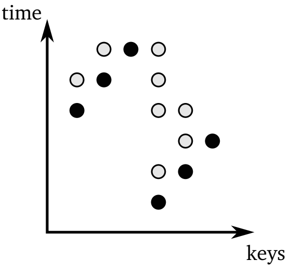

Task: Bound the cost of GREEDY

$\downarrow$

$\approx$ # of points in the GREEDY execution

Forbidden Submatrix Theory

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A useful tool since the 50s.

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Studies patterns in $0/1$-matrices.

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Studies patterns in ~~0/1-matrices~~
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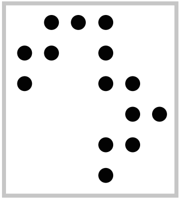

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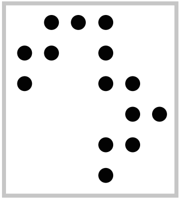

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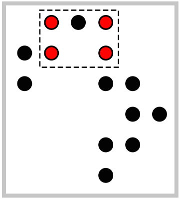

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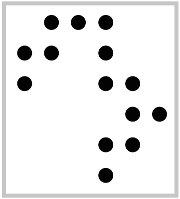

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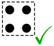

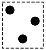

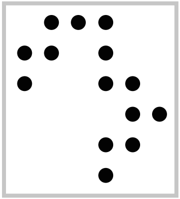

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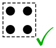

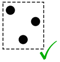

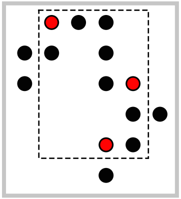

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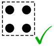

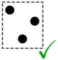

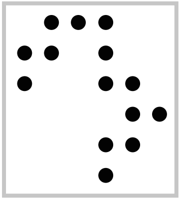

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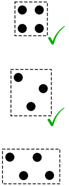

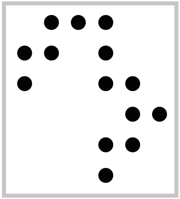

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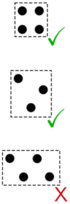

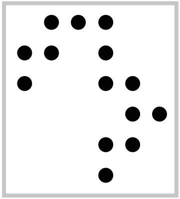

Studies patterns in ~~0/1-matrices~~
points on a grid

How many points can we have while avoiding some pattern?

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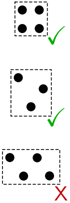

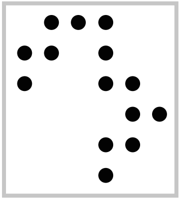

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Theorems of the form:

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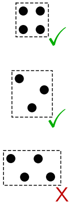

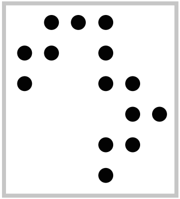

Theorems of the form:

M is a set of points on the n-by-n grid **avoiding** pattern P

Forbidden Submatrix Theory

A useful tool since the 50s.

[Zarankiewicz 1951] [Kővári, Sós, Turán '55] [Bollobás, Erdős '78] [Hart, Sharir '86] [Bienstock, Győri, '91] [Füredi, Hajnal '92] [Marcus, Tardos '04] [Pettie '10]

Studies patterns in ~~0/1-matrices~~
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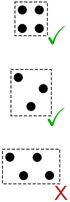

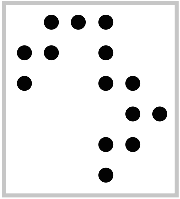

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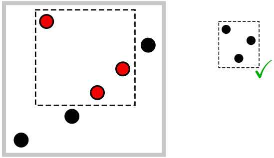
Subsumes the pattern-avoidance mentioned earlier:

134562 contains 231



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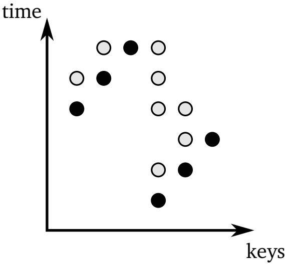
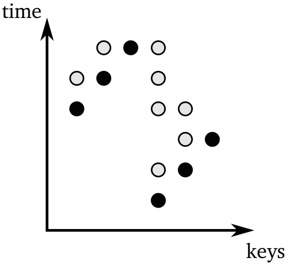
… back to Greedy

... back to GREEDY



... back to GREEDY

We bound the cost of GREEDY using forbidden submatrix theory.



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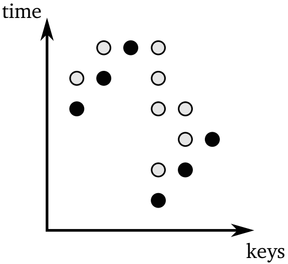
A first (WRONG) conjecture:



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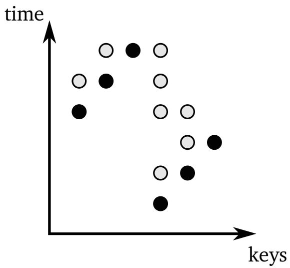

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If X avoids $\begin{pmatrix} \bullet & & \\ & \bullet & \\ & & \bullet \end{pmatrix}$

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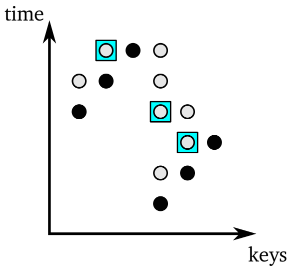

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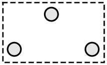
If GREEDY execution contains the pattern:



A (correct) **Lemma:**

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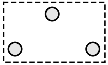
there must be an access point inside



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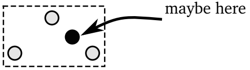

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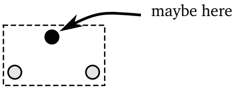

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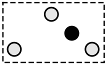

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proof very easy (but skipped).

We call this the **input-revealing** property of Greedy.

Consequence:

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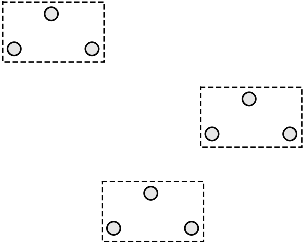

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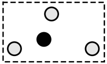

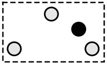

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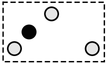

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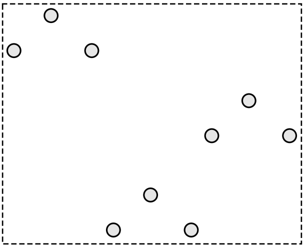

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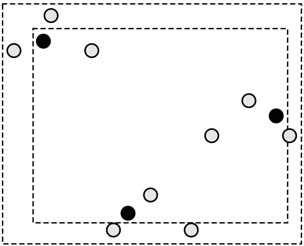

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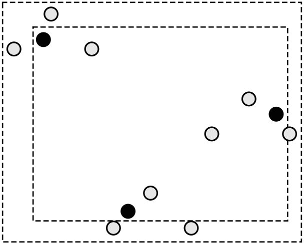

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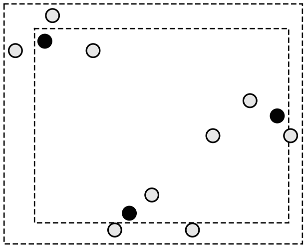

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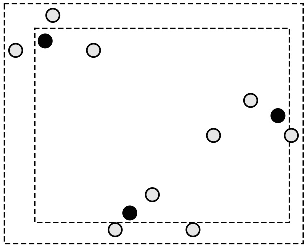

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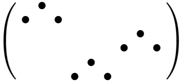
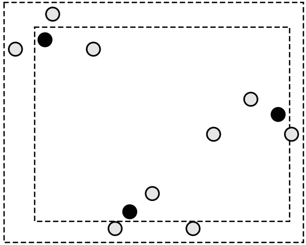
$\implies$ Greedy execution avoids 

$\implies$ cost of Greedy on X is at most $n \cdot 2^{\mathsf{poly}(\alpha(n))}$

using [Klazar '00] [Keszegh '09] [Pettie '15]

We bound the cost of Greedy using forbidden submatrix theory.



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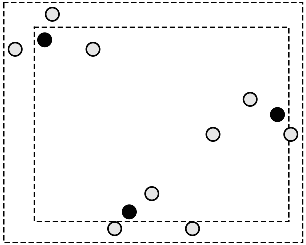

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$\rightarrow$ for various **special cases** we prove stronger bounds, i.e. $O(n)$

proofs more difficult

A different application of the technique...

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Independent Rectangle bound [Demaine et al. '09] [Wilber '89]

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$\rightarrow$ Lower bound on the cost of **any** BST algorithm

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We show:
If X **avoids** P, then **IR-bound** for X is $O(n)$, for any constant-sized P.

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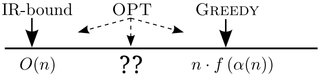

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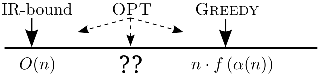

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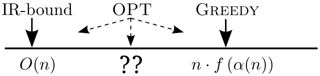

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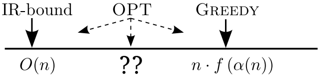

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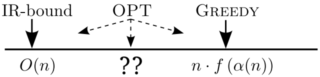

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- ? GREEDY is not $O(1)$-competitive

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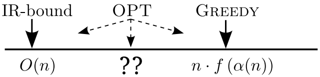

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- ? GREEDY is in fact linear on all pattern-avoiding input
- ? GREEDY is not $O(1)$-competitive
- ? Conjecture is false (IR-bound not tight)

Conclusion:

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On inputs that avoid an arbitrary pattern, GREEDY is linear*

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For traversal conjecture, GREEDY is linear*†

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Prove traversal conjecture unconditionally for an online algorithm.

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Prove traversal conjecture unconditionally for an online algorithm.
Even for preorder sequence of a **path**

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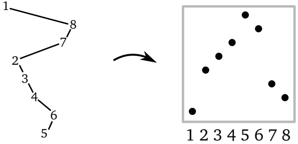

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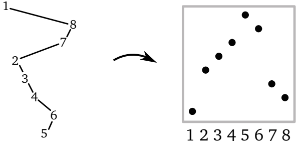

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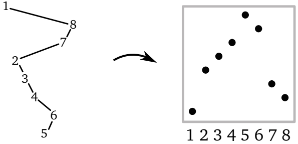

Open Question 2
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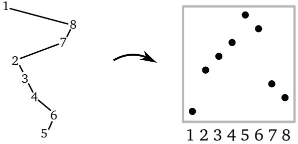

Open Question 2

Prove $o(\log(n))$-competitiveness for GREEDY or Splay Tree, or
$o(\log\log(n))$-competitiveness for any algorithm.